INDIANA UNIVERSITY

University Graduate School 2008-2009 Academic Bulletin

University Graduate School
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Mathematics

College of Arts and Sciences Bloomington

Chairperson
Professor James Davis*

Departmental E-mail
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Departmental URL
www.math.indiana.edu/programs

Graduate Faculty

(An asterisk [*] denotes membership in the University Graduate School faculty with the endorsement to direct doctoral dissertations.)

College Professor
Roger Meyer Temam*

Distinguished Professor
Ciprian Foias* (Emeritus)

William H. Boucher Professor
Vladimir Touraev*

Professors
Steen Anne Andersson*, Goro Azumaya* (Emeritus), S. Thomas Bagby* (Emeritus), Eric D. Bedford*, Grahame Bennett*, Hari Bercovici*, Rabi Bhattacharya* (Emeritus), Richard C. Bradley*, John Brothers* (Emeritus), Arlen Brown* (Emeritus), John Challifour* (Physics), Jiri Dadok*, James F. Davis*, Vinay Deodhar*, Allan L. Edmonds*, Robert T. Glassey*, Victor W. Goodman* (Emeritus), Darrell Eugene Haile*, David C. Hoff*, Jan Jaworowski* (Emeritus), Michael S. Jolly*, Nets Hawk Katz*, Paul A. Kirk*, Jee Heub Koh*, Michael J. Larsen*, Andrew Lenard* (Emeritus, Physics), Charles Livingston*, Morton Lowengrub* (Emeritus), Valery Lunts*, Russell Lyons*, Daniel P. Maki* (Emeritus), Lawrence S. Moss*, Kent E. Orr*, Sergey Ivanovich Pinchuk*, Madan Puri* (Emeritus), Billy Rhoades* (Emeritus), Jacob Rubinstein*, George Springer* (Emeritus, Computer Science), Joseph Stampfli* (Emeritus), Peter J. Sternberg*, Maynard Thompson* (Emeritus), Alberto Torchinsky*, Lanh Tat Tran*, Shouhong Wang*, Zhenghan Wang, William Ziemer* (Emeritus), Kevin R. Zumbrun*

Associate Professors
Scott W. Brown*, Mihai Ciucu*, Marlies Gerber*, Elizabeth Housworth*, Christopher Martin Judge*, Ayelet E. Lindenstrauss*, Robert MacKenzie* (Emeritus), Michael Mandell*, Kevin Michael Pilgrim*, Ji-Ping Sha*, Bruce Michael Solomon*, Matthias Weber*, William H. Wheeler*

Assistant Professors
Christopher G. Connell*, Ciprian Demeter*, David Fisher*, Patricia Lynn Hersh*

Director of Graduate Studies
Associate Professor Ji-Ping Sha*, Rawles Hall 130, (812) 855-3700

Degrees Offered

Master of Arts, Master of Arts for Teachers, and Doctor of Philosophy

Special Departmental Requirements

(See also general University Graduate School requirements.)

Admission Requirements
Undergraduate mathematics major or its equivalent.

Area Options
In order to describe the various concentration requirements, the Department of Mathematics has classified its courses into three areas: pure mathematics, applied mathematics-numerical analysis, and probability-statistics. Each of these areas is further subdivided into fields. Pure mathematics is subdivided into analysis (real and complex), algebra, topology and geometry, and logic and foundations. Applied mathematics-numerical analysis is subdivided into applied mathematics, mechanics, and numerical analysis. Probability-statistics is subdivided into probability and statistics.

Students with a strong interest in Physics might consider the Ph.D. program in Mathematical Physics.

Master of Arts Degree

Course Requirements
A total of 30 credit hours, of which 18 credit hours must be mathematics courses at the 500 to 700 level, excluding M553, M555, M556, M595, M596, and M599. The total course work submitted must include courses satisfying one of the following options and must be approved by the director of graduate studies.

1. Applied Mathematics Option. Courses including M511, M513, and at least two of the following: M540, M541,

M544, M545, M571, M572. In addition, the student must take at least one 3 credit hour graduate course outside the Department of Mathematics.
2. Pure Mathematics Option. At least 6 credit hours in each of three of the following groups: algebra, analysis, applied mathematics and numerical analysis, logic and foundations, probability and statistics, topology and geometry, outside area (biology, psychology, computer science, economics, chemistry, physics, and others).
3. Statistics Option. M511, M563, M566, and at least two of the following: M564, M567, M568, Economics E671, or any other approved course outside the Department of Mathematics.

Master of Arts for Teachers Degree

Course Requirements
A total of 36 credit hours, with at least one 3 credit hour course in each of the following groups: algebra, analysis, probability and statistics, topology and geometry, applied mathematics and numerical analysis.

Restrictions
Only Department of Mathematics graduate courses numbered 400 or higher count toward the 20 credit hours required in the major; up to 6 credit hours of courses below 400 accepted, with consent of the director of graduate studies, in partial fulfillment of the remaining 16 credit hours.

Doctor of Philosophy Degree

Course Requirements
The following course requirements are designed to provide the broad background needed for the successful pursuit of research leading to the dissertation. Students must complete 36 credit hours in mathematics at the 500, 600, or 700 level, excluding M553, M555, M556, M595-M596 and M599, and, in addition, must complete 2 credit hours in M599. Their program of study will depend upon their background and interests. Students should formulate a program in consultation with their faculty advisor. The total course work submitted for the degree must satisfy one of the options below. Reading courses may not be used to satisfy the requirements of these options unless they are specifically approved by the graduate director. A dissertation is required.

1. Applied Mathematics Option. Students must complete M511-M512, M513, and M540-M541. Students must also complete either 6 credit hours of graduate credit outside the Department of Mathematics in an area conducive to mathematical treatment and approved by the student's advisor, or 12 credit hours of graduate credit that is cross-listed with the Department of Physics. In addition, students must complete 6 credit hours of graduate credit in each of two of the following groups:
   a. Algebra, and logic and foundations
   b. Topology and geometry
   c. Numerical analysis, probability, and statistics
2. Pure Mathematics Option. Students must complete 6 credit hours of graduate credit in five of the following groups, including 6 credit hours of 500-level courses in two of groups 1-4, and 6 credit hours of 500-level courses in another of groups 1-6.
   a. Analysis
   b. Algebra
   c. Topology and geometry
   d. Logic and foundations
   e. Probability and statistics
   f. Applied mathematics and numerical analysis
   g. Outside and miscellaneous courses (cryptography, quantum computing, financial mathematics, computer science, economics, and physics are commonly used, but others may also be appropriate). Any courses in this category must be approved by your advisor and the director of graduate studies.

3. Statistics Option. Each student's program should be arranged to include work in mathematics, mathematical statistics, probability, and the application of statistics to some particular field. Students must complete 6 hours of graduate credit in other departments in courses approved by their advisor, as well as M511-M512, M563-M564, M566-M567, and 6 hours of graduate credit in each of two of the following groups:
   a. Applied mathematics and numerical analysis
   b. Algebra
   c. Complex analysis
   d. Topology and geometry
   e. Computer science

Minor
You must complete a minor in mathematics, or in some other department. If you choose to minor in another department, you must satisfy that department's requirements as described in the University Graduate School Bulletin and have that department notify the Department of Mathematics Graduate Office that you have done so.

To complete a minor in mathematics itself, there are two possible options:

1. Nine credit hour minor. This requires 9 credit hours of course work at the 500-700 level in an area (Pure Mathematics, Applied Mathematics, or Probability and Statistics) different from that of the dissertation. This area then becomes the "area for minor" while forming the Research Committee (see below).
2. Twelve credit hour minor. This requires 6 credit hours of course work at the 500-700 level in each of two fields (Analysis, Algebra, Topology and Geometry, Logic and Foundations, Applied Mathematics [Differential Equations], Mathematical Physics, Numerical Analysis, Probability, Statistics) other than that of the dissertation. One of the fields chosen must be in an area (Pure Mathematics, Applied Mathematics, Probability and Statistics) other than that of the dissertation. Any of these two fields can be chosen as the "minor field" while forming the Research Committee (see below). In addition, reading courses (e.g., M800) and

courses taken at the other universities will not satisfy the course requirements for the Ph.D. minor.

Foreign Language Requirement

Reading proficiency in one foreign language in which major research articles in mathematics are published. Acceptable languages are German, French, and Russian or another language deemed to be more relevant by the dissertation advisor. The Graduate Policy Committee of the Department of Mathematics will consider petitions for substituting other languages.

Qualifying Examinations

The Department of Mathematics qualifying exam comprises a three-tier system designed to help determine as quickly and efficiently as possible whether students have mastered basic graduate-level mathematics, exhibit the necessary abilities and self-discipline, and have prepared themselves to pursue the independent research necessary for the Ph.D. within a two- to three-year time frame.

Tier 1 (Comprehensive 400-Level Written Exams)

Ph.D. students will take a two-part written exam on 400-level algebra and analysis. The exams will be given during the week before classes begin in the fall and in the spring. New students may take either or both of the Tier 1 exams in August when they first arrive. A student is allowed to try each exam each time it is offered, but he/she must pass both exams prior to the end of the second year of study.

Syllabi, references, and sample problems for these exams are available in the Department of Mathematics graduate office.

Tier 2 (Committee Review)

Each May, a departmental committee will review the record of every student who has either:

a. Completed two years in the program without previous review, or
b. Passed the Tier 1 exams on entrance to the program and elects the review at the end of the first year.

The review committee will decide which students may continue toward Ph.D. candidacy. The committee's considerations will include:

a. Performance on the Tier 1 exams.
b. Performance in 500-level course work.
c. A report from the student's academic advisor (see below).
d. Written personal statement by student.
e. Student's performance of assistantship duties.

As indicated above, students can accelerate their progress in the program by passing the Tier 1 exams on entrance into the program and electing the Tier 2 review at the end of their first year. The review committee will treat this as favorable for a student's case. Students who do not get a recommendation to continue will be encouraged to complete the M.A. degree. If they have financial support at the time of review, they will be entitled to at least one additional semester of support in order to do so.

Tier 3 (Oral Exam)

After the Tier 2 review, students must arrange and pass a Qualifying Oral Exam before October of their fourth year. The student will seek the direction of a faculty member as a scientific advisor a "tentative Ph.D. advisor" for this exam. The faculty member will assign a reading list consisting of texts and research-level papers; this material will comprise the major topic of the exam. The student will also propose a minor area, to be approved by the director of graduate studies. If and when the scientific advisor feels the student is ready for the exam, the advisor will arrange for a three-member faculty committee to administer the exam. These exams are projected to last approximately two hours, and one of the committee members must be qualified to examine the student in the minor area, where the student must demonstrate 500-level mastery. In order to pass the exam, the student must:

a. Demonstrate a level of mathematical ability and maturity sufficient for successfully undertaking a Ph.D. dissertation (normally in the major area of the exam), and
b. Identify a faculty member willing to serve as Ph.D. advisor. This will typically, but not necessarily, be the faculty member who organized the exam.

Two aspects of the graduate program that directly support this three-tiered system bear mention:

1. A faculty advisor will be assigned to every entering student. The advisor will be expected to follow the student's progress in both course work and instructional duties, and to write a short report for the Tier 2 review committee at the appropriate time.
2. Grades in 500-level courses will be given and evaluated according to the following guidelines:
   a. A grade of A means that, based on the student's work in that course, the instructor believes the student will succeed in being admitted to Ph.D. candidacy.
   b. A grade of B means that the student's work in that course is satisfactory, but the instructor has reservations (based on that work) about the student's ability to be admitted to candidacy.

All students must maintain at least a B average in their 500-level course work, in accordance with currently published departmental and university guidelines. Tier 1 exams are administered at the end of the week before classes begin in August and January, and will be allotted at least four hours (the intention is that time should not be a serious constraint).

Ph.D. Minor in Mathematics

Doctoral students in other departments may complete a minor in mathematics by satisfying one of the following options: (1) 9 credit hours of mathematics courses at the 400 level or above, or (2) M343-M344 and 6 credit hours of mathematics courses at the 400 level or above. Reading courses (e.g., M800) and courses taken at other universities will not satisfy the course requirements for the Ph.D. minor.

Courses

Students are advised to begin their study of a field with 400-level courses, unless their preparation in that field has been very good. M.A.T. students in mathematics, or M.A., M.S., or Ph.D. students in other departments, may receive graduate credit for any 400-level course. Candidates for the M.A. or Ph.D. in mathematics should note that some 400-level courses do not satisfy certain degree requirements (see footnotes). In the following list, the middle digit of the course number indicates the field of mathematics: x0y, algebra; x1y, analysis; x2y, topology; x3y, geometry; x4y, applied mathematics; x5y, mechanics; x6y, probability and statistics; x7y, numerical analysis; x8y, history and foundations.

M403-M404 Introduction to Modern Algebra I-II (3-3 cr.)
S403-S404 Honors Course in Modern Algebra I-II (3-3 cr.)
T403 Modern Algebra for Secondary Teachers (3 cr.)[1]

M405 Number Theory (3 cr.) P: M212 (Bloomington campus only)[1]

M409 Linear Transformations (3 cr.)[2]
M413-M414 Introduction to Analysis I-II (3-3 cr.)
M415 Elementary Complex Variables with Applications (3 cr.)
M420 Metric Space Topology (3 cr.)[1]
M425 Graph Network Theory and Combinatorial Analysis (3 cr.)
M435 Introduction to Differential Geometry (3 cr.)
M436 Introduction to Geometries (3 cr.)
M441-M442 Introduction to Partial Differential Equations with Applications I-II (3-3 cr.)
M447-M448 Mathematical Models and Applications I-II (3-3 cr.)[1]
M463-M464 Introduction to Probability Theory I-II (3-3 cr.)
M466 Introduction to Mathematical Statistics (3 cr.)

M471-M472 Numerical Analysis I-II (3-3 cr.) P: M301 or M303, M311, M343, and knowledge of a computer language such as Fortran, C, or C++. (Students with other programming backgrounds should consult the instructor.)

M482 Mathematical Logic (3 cr.)
M490 Problem Seminar (3 cr.)

T490 Topics for Elementary Teachers (3 cr.) P: T103 or equivalent. Development and study of a body of mathematics specifically designed for experienced elementary teachers. Examples include probability, statistics, geometry, and algebra. Open only to graduate elementary teachers with consent of the instructor. (Does not count toward the area requirements for the M.A. and Ph.D. degrees in mathematics.)

M501 Survey of Algebra (3 cr.) P: M403-M404. Groups with operators: Jordan-Holder theorem. Sylow theorems. Rings: localization of rings; Chinese remainder theorem. Modules over principal ideal domains: invariants. Fields: algebraic closure; separable and inseparable algebraic extensions; Galois theory; finite fields.

M502 Commutative Algebra (3 cr.) P: M501. Field theory: transcendental extensions; separable extensions; derivations. Modules: Noetherian and Artinian modules. Primary modules; primary decomposition; Krull intersection theorem. Commutative rings: height and depth of prime ideals. Integral extensions. Notions of algebraic geometry: algebraic sets; Hilbert Nullstellensatz; local rings.

M503 Noncommutative Algebra (3 cr.) P: M501. Simple and semisimple modules; density theorem; Wedderburn-Artin theorem. Simple algebras: automorphisms; splitting fields; Brauer groups. Representations of finite groups: characters; induced characters; applications.

M505-M506 Basic Number Theory I-II (3-3 cr.) P: M403-M404. Congruence, units modulo n, lattices and abelian groups, quadratic residues, arithmetic functions, diophantine equations, Farey fractions, continued fractions, partition function, the Sieve method, density of subsets of integers, zeta function, the prime number theorem.

M507-M508 Introduction to Lie Algebras and Lie Groups (3 cr.) P: M403-M404, and M409 or M501. Nilpotent, solvable, and semisimple Lie algebras, PBW theorem, Killing form, Cartan subalgebras, root systems, Weyl group, classification and representations of complex semisimple Lie algebras, maximal weight modules; correspondence between real Lie algebras and Lie groups, compact Lie groups, complex and real semisimple Lie groups, symmetrical spaces.

M509 Representations of Finite Groups (3 cr.) P: M409 or equivalent. Groups, subgroups. Homomorphisms, isomorphisms. Transformation groups. The orthogonal and Euclidean groups O(3) and E(3). Symmetry and discrete subgroups of E(3). Crystallographic groups. Group representations. Reducible and irreducible representations. Group characters and character tables. Representations of the symmetric groups. Young tableaux. Symmetry classes of tensors.

M511-M512 Real Variables I-II (3-3 cr.) Sets and functions; cardinal and ordinal numbers; metric spaces; limits and continuity; function spaces and linear functionals; set functions; kinds of measures, integration; absolute continuity; convergence theorems; differentiation and integration.

M513-M514 Complex Variables I-II (3-3 cr.) Algebra, topology, and geometry of the complex plane; analytic functions; conformal mapping; Riemann surfaces; Cauchy's theorem and formula; convergence theorems; infinite series and products; Riemann mapping theorem.

M518 Fourier Analysis (3 cr.) The course will cover basic facts of Fourier series and orthogonal sets of functions, Fourier transforms, and applications. Different convergence properties of the Fourier, Haar, and Sturm-Liouville expansions will be considered. As time permits, applications to discrete and fast Fourier transforms, and wavelets, will be discussed.

M521-M522 Topology I-II (3-3 cr.) Point-set topology, including connectedness, compactness, separation properties, products, quotients, metrization, function spaces. Elementary homotopy theory including fundamental group and covering spaces. Introduction to homology theory with applications such as the Brouwer Fixed Point Theorem.

M529 Introduction to Differential Topology (3 cr.) P: M303, M413, or equivalent. Derivatives and tangents; Inverse Function Theorem; immersions and submersions; Sard's Theorem. Manifolds; imbedding manifolds. Applications: intersections and degrees (mod 2); Brouwer Fixed Point Theorem. Orientation of manifolds; Euler characteristic; Hopf Degree Theorem.

M533-M534 Differential Geometry I-II (3-3 cr.) Differentiable manifolds, multilinear algebra, and tensor bundles. Vector fields, connections, and general integrability theorems. Riemannian manifolds, curvatures, and topics from the calculus of variations.

M540-M541 Partial Differential Equations I-II (3-3 cr.) P: M441-M442 or equivalent. Introduction to distributions, Sobolev spaces, and Fourier transforms; elliptic equations, Hilbert space theory, potential theory, maximum principle; parabolic equations and systems, characteristics, representations of solutions, energy methods; applications and examples.

M542 Nonlinear Partial Differential Equations (3 cr.) Introduction to an array of topics in linear and nonlinear PDE including elements of calculus of variations and applications to nonlinear elliptic PDE, systems of conservation laws, semi-group theory, reaction-diffusion equations, Schauder theory, Navier-Stokes equations, bifurcation theory.

M544-M545 Ordinary Differential Equations I-II (3-3 cr.) P: M413-M414 or consent of instructor. Existence, uniqueness, continuous dependence; linear systems, stability theory, Floquet theory; periodic solutions of nonlinear equations; Poincaré-Bendixson theory, direct stability methods; almost periodic motions; spectral theory of nonsingular and singular self-adjoint boundary-value problems; two-dimensional autonomous systems; the saddle-point property; linear systems with isolated singularities.

M546 Control Theory (3 cr.) Examples of control problems; optimal control of deterministic systems: linear and nonlinear. The maximal principle: stochastic control problems.

M548 Mathematical Methods for Biology (3 cr.) P: M414, M463. Deterministic growth models. Birth-death processes and stochastic models for growth. Mathematical theories for the spread of epidemics. Quantitative population genetics.

M551 Markets and Multi-Period Asset Pricing (3 cr.) P: M463, M345, or equivalent. The concepts of arbitrage and risk-neutral pricing are introduced within the context of dynamic models of stock prices, bond prices, and currency exchange rates. Specific models include multi-period binomial models, Markov processes, Brownian motion, and martingales.

M553 Cryptography (3 cr.)*** P: M301 or M303. Covers encryption and decryption in secure codes. Topics include: cryptosystems and their cryptanalysis, Data Encryption Standard, differential cryptanalysis, Euclidean algorithm, Chinese remainder theorem, RSA cryptosystem, primality testing, factoring algorithms, ElGamal cryptosystem, discrete log problem, other public key cryptosystems, signature schemes, hash functions, key distribution, and key agreement. Credit not given for both M553 and M453.

M555-M556 Quantum Computing I-II (3-3 cr.)*** Covers the interdisciplinary field of quantum information science for graduate students in computer science, physics, mathematics, philosophy, and chemistry. Quantum information science is the study of storing, processing, and communicating information using quantum systems.

M557-M558 Introduction to Dynamical Systems and Ergodic Theory (3 cr.) Iteration of mappings, flows. Topological, smooth, measure-theoretic, and symbolic dynamics. Recurrence and chaos. Ergodic theory, spectral theory, notions of entropy. Low-dimensional phenomena; hyperbolicity; structural stability and rigidity. Application to number theory, data storage, Internet search and Ramsey theory.

M560 Applied Stochastic Processes (3 cr.) P: M343, M463, or consent of instructor. Simple random walk as approximation of Brownian motion. Discrete-time Markov chains. Continuous-time Markov chains; Poisson, compound Poisson, and birth-and-death chains; Kolmogorov's backward and forward equations; steady state. Diffusions as limits of birth-and-death processes. Examples drawn from diverse fields of application.

M563-M564 Theory of Probability I-II (3-3 cr.) P: M463, M512; or consent of instructor. Basic concepts of measure theory and integration, axiomatic foundations of probability theory, distribution functions and characteristic functions, infinitely divisible laws and the central limit problem, modes of convergence of sequences of random variables, ergodic theorems, Markov chains, and stochastic processes.

M566-M567 Mathematical Statistics I-II (3-3 cr.) P: M466, M512; or consent of instructor. Modern statistical inference, including such topics as sufficient statistics with applications to similar tests and point estimates, unbiased and invariant tests, lower bounds for mean square errors of point estimates, interval estimation, linear hypothesis, analysis of variance, sequential analysis, decision functions, and nonparametric inference.

M568 Time Series Analysis (3 cr.) P: M466 or consent of instructor. Autocovariance, power spectra, windows, prewhitening, aliasing, variability and covariability, rejection filtering and separation, pilot estimation, cross-spectra, R-th order spectra, prediction, numerical spectrum analysis.

M571-M572 Analysis of Numerical Methods I-II (3-3 cr.) P: M441-M442 and M413-M414. Solution of systems of linear equations, elimination and iterative methods, error analyses, eigenvalue problems; numerical methods for integral equations and ordinary differential equations; finite difference, finite element, and Galerkin methods for partial differential equations; stability of methods.

M583 Set Theory (3 cr.) P: M482 or M511 or M521. Zermelo-Fraenkel axioms for set theory, well-foundedness and well-orderings, induction and recursion, ordinals and cardinals, axiom of choice, cardinal exponentiation, generalized continuum hypothesis, infinite combinatorics and large cardinals. Martin's axiom, applications to analysis and topology.

M584 Recursion Theory (3 cr.) P: One of M482, M511, M521 or CSCI C452; or consent of instructor. Classes of recursive functions, models of computation, Church's thesis, normal forms, recursion theorem, recursively enumerable sets, reducibilities, lattice of r.e. sets, jump operator, priority arguments, degrees of unsolvability, and hierarchies.

M590 Seminar (3 cr.)

M595-M596 Seminar in the Teaching of College Mathematics I-II (1-1 cr.) Methods of teaching undergraduate college mathematics. Does not count toward meeting any of the 500-level requirements toward an M.A. or Ph.D.

M599 Colloquium (1 cr.) Attendance at Department of Mathematics colloquia required. May be repeated. May not be used in fulfillment of the 36 credit hours of 500-, 600-, or 700-level course work required for the Ph.D. Also not applicable to 30 credit hours for master's degree.

M601-M602 Algebraic Number Theory I-II (3-3 cr.) P: M501-M502. Valuations, fields of algebraic functions, cohomology of groups, local and global class field theory.

M607-M608 Group Representations I-II (3-3 cr.) P: Consent of instructor. Review of abstract group theory. Representation theory of finite and infinite compact groups. Detailed study of selected classical groups. Lie groups, covering groups, Lie algebras, invariant measure and induced representations. May be taught in alternate years by members of the Departments of Mathematics and Physics; see PHYS P607.

M611-M612 Functional Analysis I-II (3-3 cr.) Fundamentals of the theory of vector spaces; Banach spaces; Hilbert space. Linear functionals and operators in such spaces, spectral resolution of operators. Functional equations: applications to fields of analysis, such as integration and measure, integral equations, ordinary and partial differential equations, ergodic theory. Nonlinear problems. Schauder-Leray fixed-point theorem and its applications to fundamental existence theorems of analysis.

M621-M622 Algebraic Topology I-II (3-3 cr.) Basic concepts of homological algebra, universal coefficient theorems for homology and cohomology, Künneth formula, duality in manifolds. Homotopy theory including Hurewicz and Whitehead theorems, classifying spaces, Postnikov systems, spectral sequences, homotopy groups of spheres. Offered every other year, alternating with M623-M624.

M623-M624 Geometric Topology I-II (3-3 cr.) P: M522. Topics in geometric topology chosen from K-theory, simple homotopy theory, topology of manifolds, fiber bundles, knot theory, and related areas. May be taken more than once. Offered every other year, alternating with M621-M622.

M630 Algebraic Geometry (3 cr.) A study in the plane, based on homogeneous point and line coordinates; a study of algebraic curves and envelopes, including such topics as invariants, singularities, reducibility, genus, polar properties, Pascal and Brainchon theorems, and Jacobian, Hessian, and Plücker formulas.

M633-M634 Algebraic Varieties I-II (3-3 cr.) Topological and algebraic properties of algebraic varieties.

M635-M636 Relativity I-II (3-3 cr.) Mathematical foundations of the theory of relativity. Lorentz groups, Michelson-Morley experiment, aberration of stars, Fizeau experiment, kinematic effects, relativistic second law of Newton, relativistic kinetic energy, Maxwell equations, ponderomotive equations. Curvature tensor and its algebraic identities, Bianchi's identity, gravitation and geodesics. Schwarzschild solution, relativistic orbits, deflection of light.

M637 Theory of Gravitation I (3 cr.) IIntroduction to the general theory of relativity, stress-energy tensor, parallel transport, geodesics, Einstein's equation, differential geometry, manifolds, general covariance, bending of light, perihelion advance. Modern cosmology: Robertson-Walker metric, equations of state, Friedmann equations, Hubble's law, redshift, cosmological constant, inflation, quintessence, cosmic microwave background, Big Bang nucleosynthesis, structure formation. May be taught in alternate years by members of the Department of Physics; see PHYS P637.

M638 Theory of Gravitation II (3 cr.) Gravitation waves, Schwarzschild geometry and black holes, Kerr metric, Reissner-Nordstrom metric, extremal black holes, Penrose diagrams, Hawking radiation, Lie derivatives, isometries and Killing vectors, variational principle and the Palatini formalism, spinors in general relativity, vierbeins, gravitation as a gauge theory, quantum gravity. May be taught in alternate years by members of the Department of Physics; see PHYS P638.

A641 Elliptic Differential Equations (3 cr.) P: M511, M513, M540, or consent of instructor. Green's identity, fundamental solutions, function theoretic methods, partition of unity, weak and strong derivatives, Sobolev inequalities, embedding theorems, Garding's inequality, Dirichlet problem, existence theory, regularity in the interior, regularity on the boundary, and selected topics.

A642 Evolution Equations (3 cr.) P: M511, M513, M540, or consent of instructor. Hyperbolic equations and systems, parabolic equations, Cauchy problems in higher dimension, method of descent, fundamental solutions and their construction, strongly continuous semigroups, analytic semigroups, uniqueness theorems in Hilbert space, fractional powers of operators, analyticity of solutions, and selected topics.

A643 Integral Equations (3 cr.) Covers the Volterra-Fredholm theory of integral equations and the abstract Riesz theory of compact operators. Other topics include ideals of compact operators, Fredholm operators, convolution equations and their relationship to Toeplitz operators, Wiener-Hopf factorization.

M647 Mathematical Physics (3 cr.) P: M541 or consent of instructor. Applications of the theory of normed linear spaces, distributions, unbounded operators in Hilbert space, and related topics to problems in mathematical physics. May be taught in alternate years by members of the Department of Physics; see PHYS P647.

M655 Mathematical Foundations of Quantum Mechanics (3 cr.) P: Consent of instructor. Philosophical and mathematical analysis of the concepts: quantum observable, compatibility, quantum state, superposition principle, symmetry. Axiomatic construction of conventional quantum mechanics. May be taught in alternate years by members of the Department of Physics; see PHYS P655.

M656-M657 Kinetic Theory and Statistical Mechanics I-II (3-3 cr.) Introduction to the classical theory and modern developments. Historical development of kinetic-statistical theories; rigorous equilibrium statistics; kinetic gas dynamics according to Boltzmann equation; kinetic theories of transport processes in liquids. May be taught in alternate years by members of the Departments of Mathematics and Physics; see PHYS P656-P657.

M658-M659 Continuum Mechanics I-II (3-3 cr.) P: Consent of instructor. Two-semester course dealing with mathematical foundations of continuum mechanics; content varies yearly; topics selected from elasticity, plasticity, or fluid mechanics and related areas.

M663 Weak Convergence of Probability Measures and Applications (3 cr.) P: M512, M564. Weak convergence of probability measures on metric spaces. Prohorov's theorem and tightness. Brownian motion. Donsker's invariance principle. Weak convergence on D [0,1]. Convergence of empirical distributions. Functional central limit theorems under dependence.

M664 Large Sample Theory of Statistics (3 cr.) P: M563, M566. Asymptotic distributions of sample moments, sample quantiles, and U-statistics; methods of estimation: maximum likelihood estimates, method of moments, L-estimators, Bayes estimators; asymptotic efficiency; likelihood ratio tests, chi-square tests, asymptotic relative efficiencies of tests; weak convergence of the empirical distribution function to a Brownian bridge and application; selection of topics from the following: large deviations, second-order asymptotic efficiency, bootstrap rank tests.

M671-M672 Numerical Treatment of Differential and Integral Equations I-II (3-3 cr.) P: M540 or consent of instructor. Finite difference methods of ordinary and partial differential equations; relaxation methods; discrete kernel functions; methods of Ritz, Galerkin, and Trefftz approximate methods for integral equations.

M680 Logic and Decidability (3 cr.) P: M584 and M404; or consent of instructor. Effective syntax and semantics of propositional and first-order logics, theory of decidability and some decidable theories, theory of undecidability and implicit definability, Gödel's theorems on incompleteness and the unprovability of consistency.

M682 Model Theory (3 cr.) P: M583, M680, and M502; or consent of instructor. Elementary equivalence, completeness and model-completeness, interpolation, preservation and characterization theorems, elementary classes, types, saturated structures, introduction to categoricity and stability.

M701-M702 Selected Topics in Algebra I-II (3-3 cr.)
M711-M712 Selected Topics in Analysis I-II (3-3 cr.)
M721-M722 Selected Topics in Topology I-II (3-3 cr.)
M731-M732 Selected Topics in Differential Geometry I-II (3-3 cr.)
M733-M734 Selected Topics in Algebraic Geometry I-II (3-3 cr.)
M741-M742 Selected Topics in Applied Mathematics I-II (3-3 cr.)

M743-M744 Selected Topics in Mathematical Physics I-II (3-3 cr.) Content varies from year to year. May be taught in alternate years by members of the Department of Physics; see PHYS P743.

M751-M752 Selected Topics in Mechanics I-II (3-3 cr.)
M761-M762 Selected Topics in Statistics and Probability I-II (3-3 cr.)
M771-M772 Selected Topics in Numerical Analysis I-II (3-3 cr.)
M781-M782 Selected Topics in Mathematical Logic (3-3 cr.)

M800 Mathematical Reading and Research (cr. arr.)** Intended primarily for graduate students who have passed the qualifying examination.

**This course is eligible for a deferred grade.
***Does not count toward the 500-level requirements.

[1] These courses do not ordinarily carry credit toward the M.A. or Ph.D. degree in mathematics. They may, however, be taken by M.A.T. students and graduate students in other departments for graduate credit.
[2] Does not count toward the area requirements for the M.A. and Ph.D. degrees in mathematics.